Bubble Behavior in Hydrodynamic Cavitation: Effect of Turbulence

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Harnessing the energy associated with cavitation for a number of physical and chemical changes has been pursued enthusiastically by sonochemists using ultrasound equipment of various sizes, shapes and forms. The effect of cavitation as observed in ultrasonic equipment has been attributed to the transient form of cavitation. Hydrodynamically generated cavities are believed to behave in a stable cavitation mode and, hence, are not very useful for the desired sonochemical effects.

This work illustrates with numerical simulations the role played by the turbulence in altering a single cavity behavior downstream of a cavitating orifice in a liquid flow. Specifically, it looks into the effect of turbulent pressure fluctuations in transforming otherwise stable cavitation into transient cavitation.

Among the practical objectives of this study is a better understanding of the bubble behavior in hydrodynamic cavitation, in particular the effect of various parameters such as intensity and frequency of eddies responsible for the turbulence downstream of the orifice that actually control the bubble dynamics. Previously, the bubble dynamics problem has been analyzed on the basis of a single bubble and a linear pressure recovery profile downstream of the orifice neglecting the turbulent pressure fluctuations (Yan et al., 1988). This approach may be adequate when the intensity of turbulence is quite low and thus pressure recovery from the vena contracta to a downstream pipe position can be approximated by a linear expression with respect to a distance downstream of the orifice. It certainly loses validity when the intensity of turbulence rises and pressure recovery is no more linear especially with changing geometry of the orifice. In this case significant alteration are likely to occur in the local pressures encountered by the vapor/bubble cavity during the passage in the flow. In this article an attempt has been made to analyze the fundamental phenomenon, numerically with some interesting results. This could be a useful basis for the effective scaleup and design of hydrodynamic cavitation reactors by describing the role of downstream turbulence on the bubble/cavity behavior.

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This analysis includes the computer simulations of modified bubble dynamics equation (Plesset, 1949) for hydrodynamic cavitation. The effects of various parameters which affect the turbulence intensity (in terms of its magnitude and frequency) and in turn its effect on the bubble behavior have been studied.

Mathematical Formulation and Algorithm of Simulation

The specific assumptions in the mathematical modeling are as follows:

(1) A single bubble or cavity has been considered in isolation.

(2) The cavitation medium is assumed to be inviscid and incompressible and simulations are terminated at a point where the instantaneous bubble size to the initial bubble size ratio (R/R_o) is 0.5 during the collapse.

(3) The relative motion between gas and liquid phase (that is, the slip velocity) and hence the friction loss has been neglected.

(4) Heat- and mass-transfer effects have been neglected due to very fast events.

Cavitation inception

A dimensionless parameter which has been widely used in the study of hydrodynamic cavitation is the cavitation number (C_i) which is defined as

$$C_i = \frac{P_2 - P_v}{\frac{1}{2}\rho v_o^2} \tag{1}$$

where P_2 is the recovered downstream pressure, and P_v is the vapor pressure of the cavitating medium at the operating temperature, ρ is the density, and v_o is the average fluid velocity at the orifice.

In the present analysis, we have assumed the cavitation number to be 1 and the simulations are on that basis only. Though in practice the cavitation inception number has been found to be a function of orifice diameter (Yan et al., 1988) and could be greater than one indicating the influence of turbulent pressure fluctuations.

Turbulence Modeling. For a pipe flow, the turbulent pressure fluctuations are due to velocity perturbations as a result of the formation of eddies. In turbulent flow, the instantaneous velocity in the x direction is given as

$$v_x = \bar{v}_x + \bar{v}'_x \tag{2}$$

where $\bar{v}_x$ is the time-averaged velocity at any point in the flowing fluid, and v'_x is the instantaneous fluctuating velocity. It is convenient to express the amplitude of the fluctuating velocities in the x direction as $(\bar{v}'_x)^2$, which is the mean of the squares of the fluctuation velocities, this being necessarily positive. Similar treatment can be given for velocity fluctuations in other directions. The turbulent kinetic energy per unit mass is thus given by

$$K.E. = -\frac{1}{2}\left[(\bar{v}'_x)^2+(\bar{v}'_y)^2+(\bar{v}'_z)^2\right] \tag{3}$$

Hence, the rate of dissipation of turbulent kinetic energy is given as

$$P_M = -\frac{1}{2}\frac{d}{dt}\left[(\bar{v}'_x)^2+(\bar{v}'_y)^2+(\bar{v}'_z)^2\right] \tag{4}$$

which for isotropic turbulence (where $\bar{v}'_x = \bar{v}'_y = \bar{v}'_z = v$) reduces to

$$P_M = -\frac{3}{2}\frac{d}{dt}(\bar{v}')^2 \tag{5}$$

For isotropic turbulence, the fluctuation velocity $\bar{v}'$ and the length scale of eddy (l) can be defined in terms of the power input per unit mass of the system P_m. It is equal to the rate of dissipation of energy per the unit mass of turbulent fluid and the relations are (Davies, 1972)

$$P_M = \frac{(\bar{v}')^3}{l} \tag{6}$$

The frequency of the velocity perturbations within the turbulent eddies is simply given as

$$f_T = \frac{\bar{v}'}{l} \tag{7}$$

In this analysis P_M has been estimated by considering the permanent pressure head loss which is a function of the ratio of the orifice to pipe diameter. The product of the head loss and the volumetric flow rate will give the energy dissipation rate due to turbulence. The rate of energy dissipation due to the pipe wall friction was found to be negligible (2 to 4% of turbulent energy dissipation rate) and hence neglected. This turbulent energy dissipation rate divided by the mass of the water in the region of pressure recovery (typically 8 pipe diameters downstream orifice) gives P_M.

For the estimation of the length scale (l) Prandtl eddy model has been used. Prandtl eddy is a medium size eddy and is also best suited for the geometry under consideration (since its size is a strong and only function of the diameter of the conduit through which the liquid and eddies flow) was selected for the analysis in this work.

According to this model the length scale is given as

$$l = 0.08\,d \tag{8}$$

where d is the diameter of the conduit through which fluid and associated cavity flows. In the region near orifice where fluid stream narrows down and flow area becomes equal to the area of orifice the eddy size is 0.08 d_o (d_o is the diameter of the orifice). Thereafter, as the flow stream expands and the flow area equals the area of pipe after full pressure recovery, the scale is 0.08 d_p (d_p is the pipe diameter). To find the frequency of turbulence (Eq. 7), the average length scales at these two extremes has been considered. Similarly, to estimate $\bar{v}'$ (Eq. 6) the same l has been considered with P_M as estimated earlier.

Bubble Dynamics. With the inclusion of surface tension and viscosity effects the equation of bubble dynamics is given by

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2 = \frac{1}{\rho}\left[P_i - P_\infty - \frac{2\sigma}{R} - \frac{4\mu}{R}\left(\frac{dR}{dt}\right)\right] \tag{9}$$

This equation known as the Rayleigh-Plesset equation forms the basis of this analysis (Plesset, 1949).

To confirm the validity of the numerical scheme, simulations of bubble behavior reported by Miksis and Ting (1984) were carried out with the current numerical scheme and these were found to be identical.

For hydrodynamic cavitation in orifice flow, the mean pressure recovery profile is assumed to be linear over which turbulent pressure fluctuations can be superimposed. The pressure P_∞, that is, the pressure at a large distance from the bubble decides the bubble behavior which will change due to the flow conditions assumed. Here the local pressure without turbulence can be estimated on the basis of linear pressure recovery as

$$P_{\text{mean}} = P_v + \frac{(P_2 - P_v)}{\tau}t \tag{10}$$

where τ is the pressure recovery time and P_{mean} is the axial pressure downstream of the orifice to be substituted for P_∞ in Eq. 9 for simulation.

The time for pressure recovery has been estimated by Newton's equations. For an assumed recovery pressure (P_2) downstream of the orifice, the velocity near the orifice can be estimated from Eq. 1 for $C_i = 1.0$. This velocity reduces with an increasing distance downstream of the orifice and reaches the mean pipe velocity when full pressure recovery takes place. Pipe velocity has been estimated by equating volumetric flow rates. Thus,

$$(d_o)^2 v_o = (d_p)^2 v_p \tag{11}$$

where d_o and d_p are the diameters of the orifice and the pipe through which the stream flows. With the knowledge of v_o and v_p the time of pressure recovery has been estimated using Newton's first and third laws of motion.

Using the mean pressure as given by Eq. 10 and the use of Bernoulli's equation (between that particular point and a point where full pressure recovery takes place) an estimation has been made of the local velocity at any particular point downstream of the orifice. The use of Bernoulli's equation for this purpose has been only at steady-state conditions as the turbulent pressure variations have not been considered (they are likely to cancel out as uniform turbulence intensity has been considered over the entire volume, which is considered for the pressure loss or energy dissipation). The turbulent velocity fluctuations have been superimposed on it by assuming a sinusoidal velocity variation in the instantaneous local velocity with a frequency estimated by Eq. 7. Thus, the new instantaneous local velocity is given as

$$v_{tn} = v_t + \bar{v}' \sin(2\pi f_T t) \tag{12}$$

where v_t is the local mean velocity and v_{tn} is a function of time with t being the numerical integration step. Use of this instantaneous velocity has been made to estimate the instantaneous local static pressure using Bernoulli's equation of the following form

$$P_t = P_v + \frac{1}{2}\rho v_o^2 - \frac{1}{2}\rho v_{tn}^2 - \Delta P \tag{13}$$

This instantaneous pressure which involves the effect of turbulent pressure fluctuations is then used instead of P_{mean} given by Eq. 10 for the solution of Eq. 9 as a variation in P_∞. The Runge-Kutta fourth-order method was adapted for the numerical solution of Eq. 9.

With the initial conditions as $R = R_o$, $dR/dt = s = 0$, at $t = 0$, the solution has been presented as radius-time or pressure-time history of a single cavity starting from its inception at vena-contracta with P_t varying from an average pressure of P_v at vena contracta until the complete recovery of the pressure to the designated value of P_2.

Results and Discussion

Results are provided of numerical solutions described earlier for the variety of operating conditions. For brevity, only representative solutions have been depicted in the subsequent figures. The parameters which have been found to affect the cavity/bubble behavior are

(1) Recovery pressure (P_2)
(2) The pipe size downstream of the orifice (d_p)
(3) Orifice to pipe diameter ratio (β)
(4) Initial cavity or bubble size (R_o)

The result of the simulations indicate the following effect on the bubble/cavity behavior.

Effect of recovery pressure

The conditions of simulations have been listed in the caption of Figure 1. It indicates that with an increase in the recovery pressure the maximum cavity size before collapse increases. The life of the bubble/cavity also increases. The increased recovery pressure increases both the intensity and frequency of turbulence affecting the cavity in a manner similar to the changing acoustic field in the ultrasonic cavitation case.

The drastic change in the cavity behavior can be seen by comparing the simulations with or without turbulence effects. With turbulence, the oscillatory stable cavitation behavior transforms into a transient cavitation, similar to the acoustic cavitation.

Effect of pipe-size downstream of orifice

The simulations conditions have been listed in the caption of Figure 2. Figure 2 indicates that with an increase in the pipe size the maximum cavity radius before collapse increases, resulting into a higher-pressure pulse. This alteration in the cavity behavior is due to the increased scale of turbulence (assumed to be a function of pipe and orifice diameters) which reduces the frequency of turbulence as the P_∞ or P_t value is only marginally altered (the effect of skin friction is small). The reduced frequency allows the cavity to grow further resulting in a larger radius before the collapse. Again, a transformation from stable to transient cavitation is evident from this figure as a result of consideration of turbulence in liquid.

Effect of β

For a fixed pipe size two different values of β have been considered for the simulation. Figure 3 indicates that with an increase in β the life of the cavity before its violent collapse increases although the maximum radius of the cavity bubble is not altered significantly. The explanation for this effect can be given as follows: A change in the orifice to pipe diameter ratio alters the permanent pressure head loss. Specifically, the permanent pressure head loss is 73% of the orifice pressure differential for a orifice to pipe diameter ratio of 0.5, while for a β of 0.75 it is around 60% of the orifice pressure differential. Therefore, intensity of the turbulence is inversely proportional to β (i.e., P_M decreases with an increasing β). Therefore, as discussed in the previous subsection the increase in β decreases turbulent intensity and hence increases the bubble life. However, the maximum bubble size reached during the transient collapse doesn't change much (specifically a change in β from 0.5 to 0.75 alters the maximum bubble size reached during bubble oscillation by less than 10%).

Effect of initial bubble radius

Simulations were performed for two different bubble sizes (viz., 10, 100 microns) under otherwise similar conditions. The parameters of simulations are listed in the caption of Figure 4. The sizes of the cavities were chosen to cover the widest possible size distribution expected in hydrodynamic flow situation (Yan et al., 1990).

It is evident from the results that the ratio R/R_o before collapse is higher for smaller bubbles. Therefore, it is obvious that the contribution to the cavitational effect due to the pressure pulse is higher due to smaller bubbles, since the

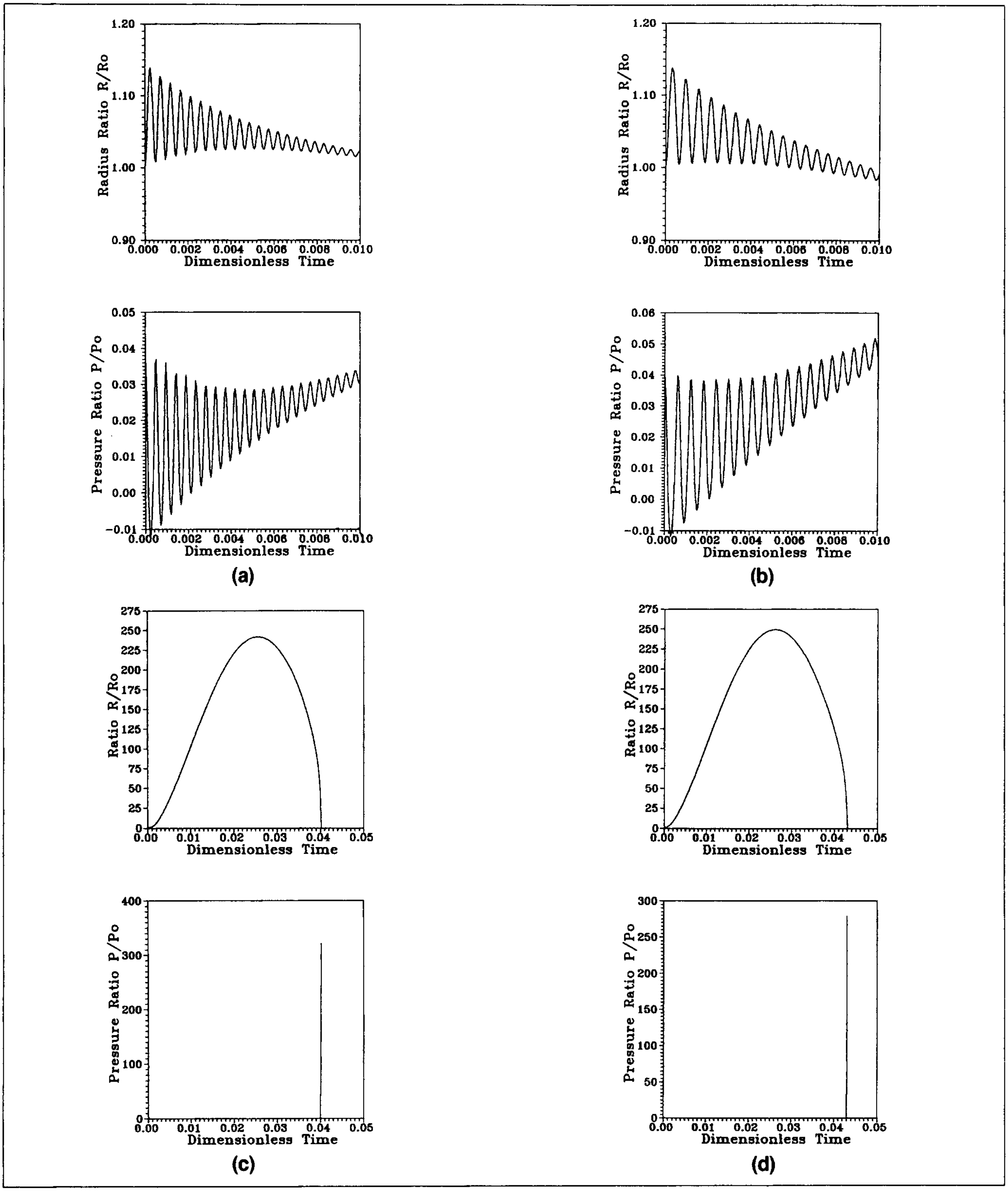

Figure 1. Radius history and pressure pulses of bubble oscillations under turbulent and nonturbulent conditions: Effect of recovery pressure.

Parameters for simulations:

β	P_2 atm	d_p in.	d_o in.	R_o μm	f_T kHz	$1/\tau$ kHz	Figure No. without Turbulence	Figure No. with Turbulence
0.5	3	1.5	0.75	10	1.492	0.05185	1a	1c
0.5	5	1.5	0.75	10	1.926	0.06713	1b	1d

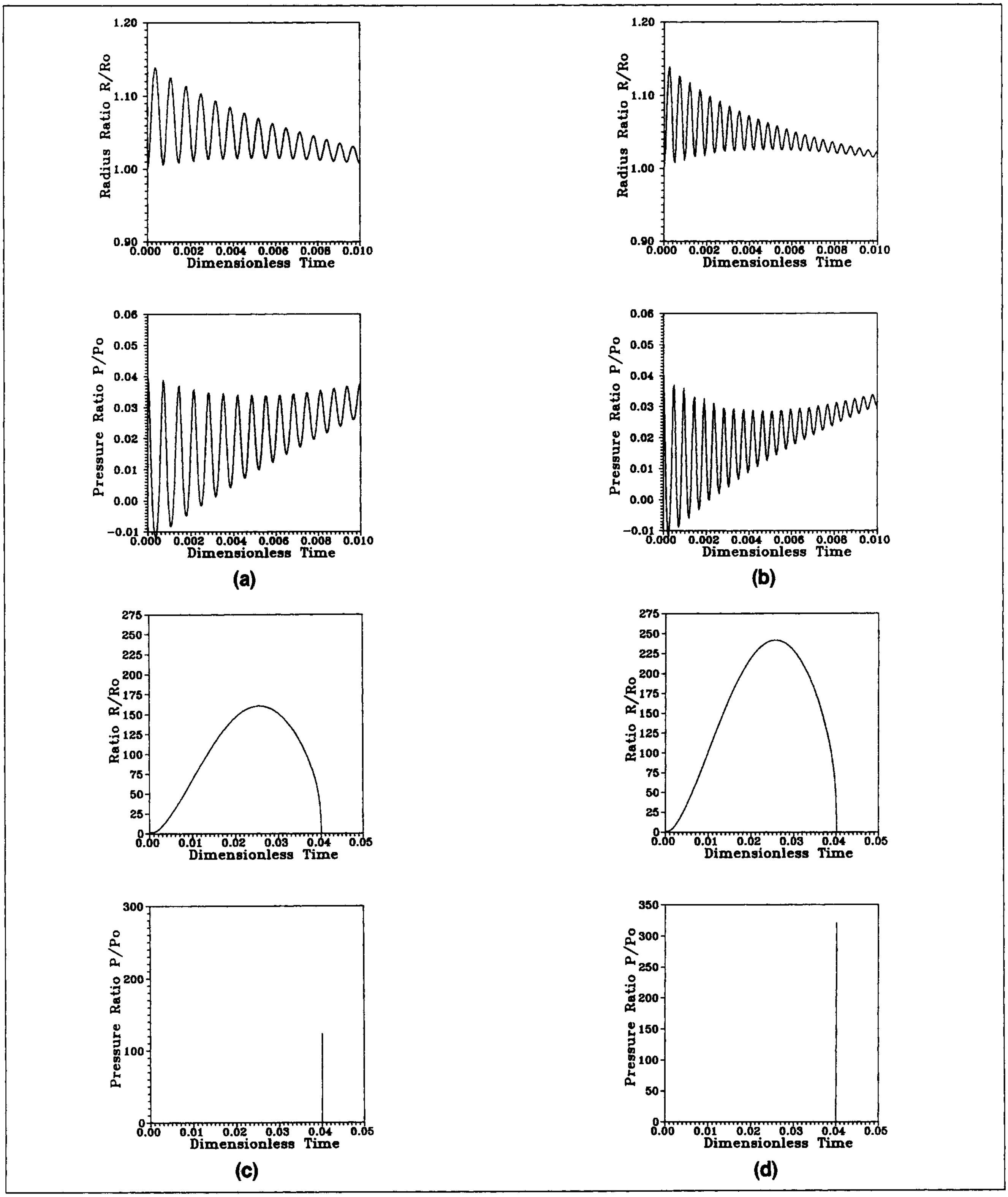

Figure 2. Radius history and pressure pulses of bubble oscillations under turbulent and nonturbulent conditions: Effect of pipe size downstream of the orifice.

Parameters for simulations:

β	P_2 atm	d_p in.	d_o in.	R_o μm	f_T kHz	$1/\tau$ kHz	Figure No. without Turbulence	Figure No. with Turbulence
0.5	3	1	0.5	10	2.239	0.07778	2a	2c
0.5	3	1.5	0.75	10	1.492	0.05185	2b	2d

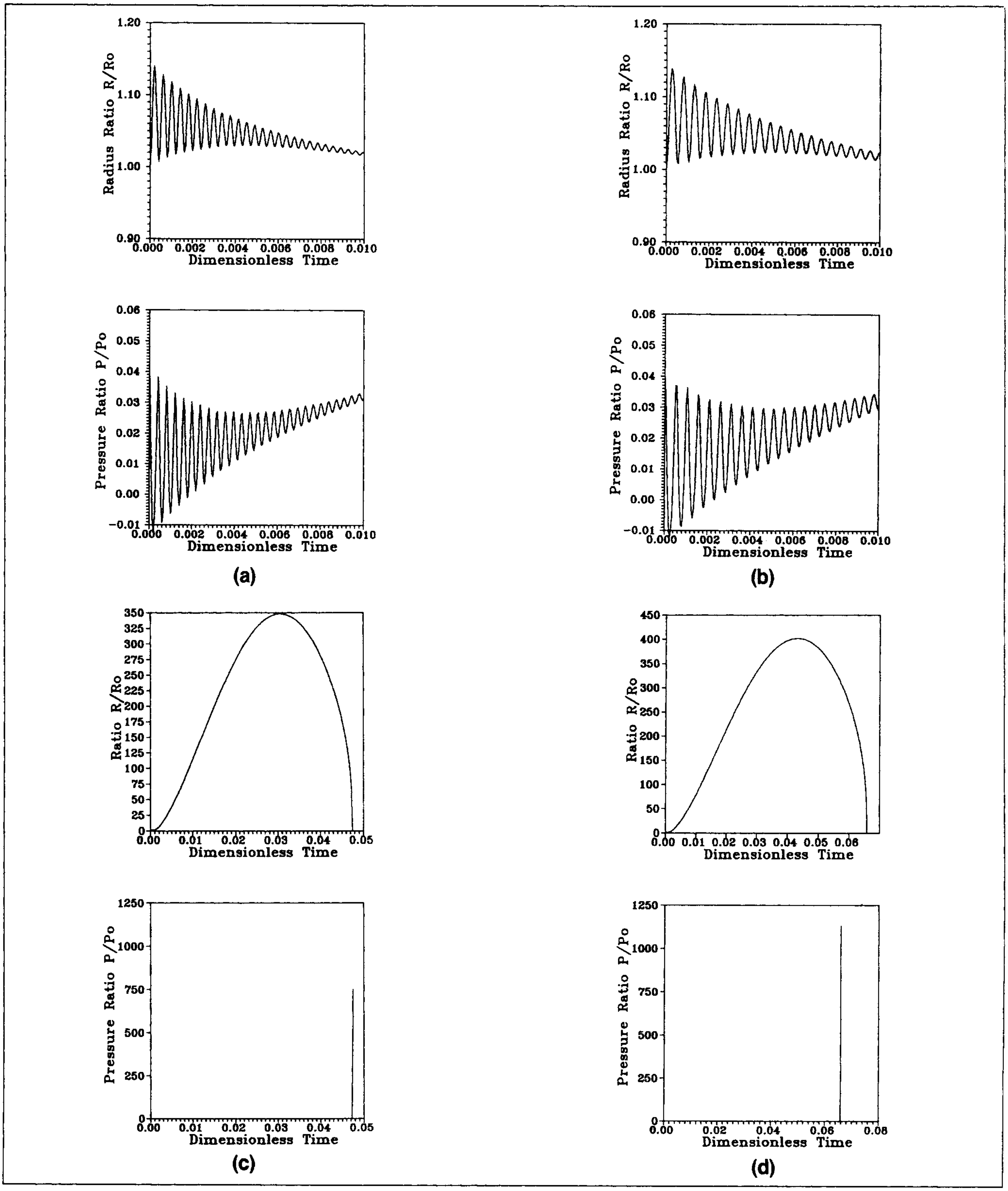

Figure 3. Radius history and pressure pulses of bubble oscillations under turbulent and nonturbulent conditions: Effect of orifice to pipe diameter ratio.

Parameters for simulations:

β	P_2 atm	d_p in.	d_o in.	R_o μm	f_T kHz	$1/\tau$ kHz	Figure No. without Turbulence	Figure No. with Turbulence
0.6	3	2	1.2	10	1.056	0.04391	3a	3c
0.75	3	2	1.5	10	0.961	0.05693	3b	3d

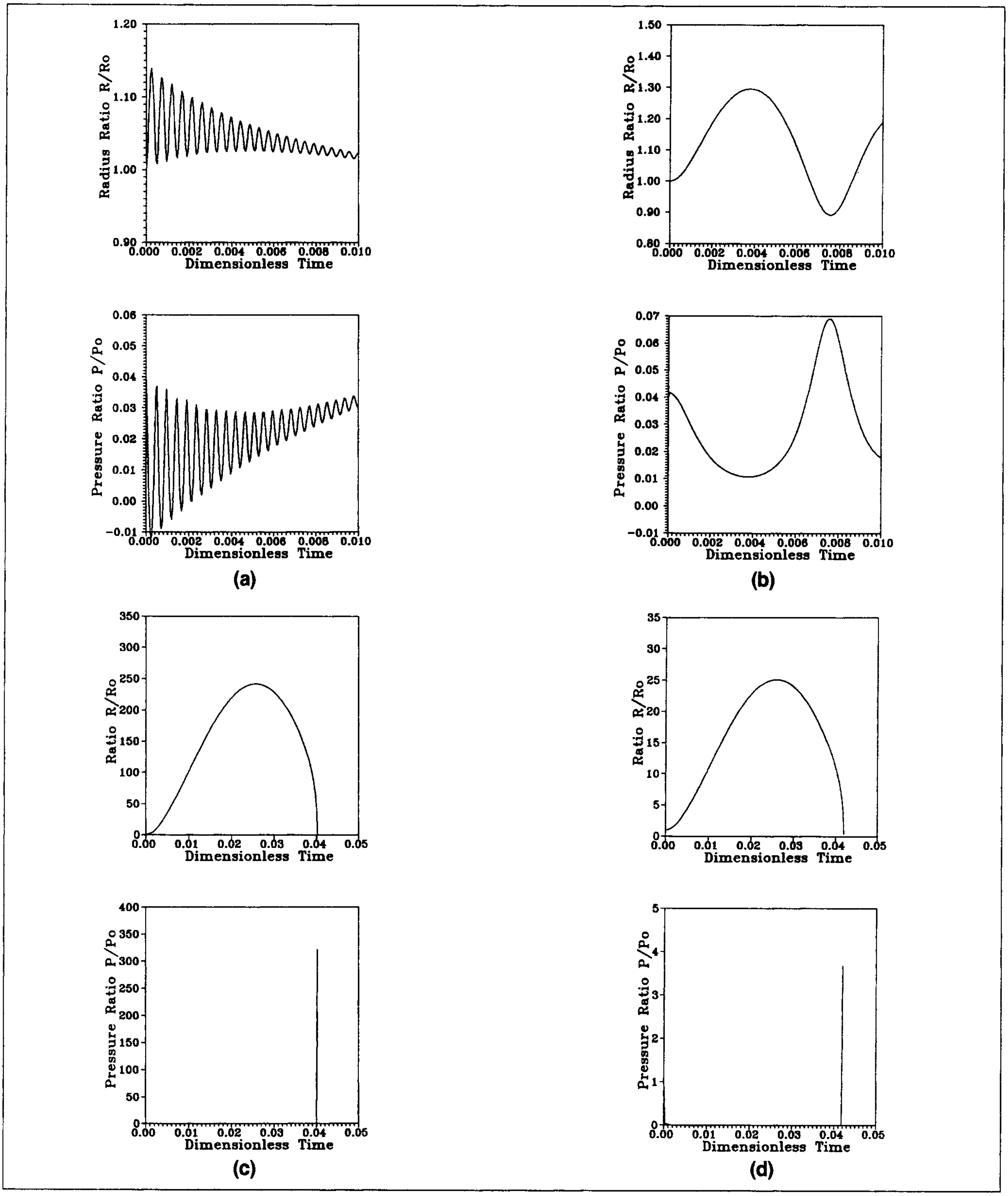

Figure 4. Radius history and pressure pulses of bubble oscillations under turbulent and nonturbulent conditions: effect of initial bubble radius.

Parameters for simulations:

β	P_2 atm	d_p in.	d_o in.	R_o μm	f_T kHz	$1/\tau$ kHz	Figure No. without Turbulence	Figure No. with Turbulence
0.5	3	1.5	0.75	10	1.492	0.05185	4a	4c
0.5	3	1.5	0.75	100	1.492	0.05185	4b	4d

magnitude of the pressure pulse produced by these bubbles during collapse is proportional to the ratio of R/R_o, R being the maximum radius before the violent collapse.

Conclusion

Simulations have been performed to assess the influence of various flow and geometrical parameters on the turbulence intensity and thus on the bubble behavior in hydrodynamic orifice flow cavitation. In spite of the assumptions made in the algorithm, the simulations do indeed represent some interesting physical phenomenon.

It was found that the bubble behavior changes drastically under the turbulent flow conditions. As compared to the oscillatory behavior under nonturbulent conditions; the bubble behavior under turbulent conditions is transient and resembles the behavior of a cavity under acoustic cavitation. This is an interesting result and opens up a number of possibilities for the design of cavitating reactors for the so-called sonochemical effects. It means that acoustic cavitating conditions as generated in ultrasonic equipments can be more simply generated in hydrodynamic flow situations by manipulating turbulence levels. This conclusion has been given credence if one compares the identical metal erosion rates obtained under acoustic and hydrodynamic cavitating conditions. To obtain, identical metal erosion rates, the pressure pulses produced by collapsing cavities need to be of similar magnitude which can only be obtained with transient cavitation (with turbulence) and not by stable cavitation (without turbulence) (Hansson et al., 1977).

Independent studies (Shirgaokar et al., 1997) carried out on a high-pressure homogenizer having geometry similar to that of a hydrodynamic cavitation reactor have also confirmed this change in cavity behavior from stable to transient.

The model reaction of the decomposition of the aqueous KI solution used by Prasad Naidu et al. (1994) was attempted on high-pressure homogenizers. It was discovered that by manipulating the discharge pressures and hence the fluid velocities through the valve gap it was possible to decompose KI in the aqueous solution releasing iodine which was measured spectrophotometrically. It is generally agreed that for the decomposition of KI in aqueous solutions, transient cavity behavior is necessary (Prasad Naidu et al., 1994). Thus, decomposition of KI in aqueous solutions as observed in the high-pressure homogenizer confirms the existence of transient cavity behavior. This behavior as predicted by the earlier discussions is only possible under turbulent flow conditions (homogenizer valve Reynolds number $> 4,000$) and thus can be treated as a preliminary evidence of the transformation of cavity behavior from oscillatory to transient (No. KI decomposition was observed at $Re < 4,000$).

The variables studied were found to affect mainly two aspects of the bubble behavior: (i) Maximum bubble radius reached during the oscillations, and (ii) bubble life.

In hydrodynamic cavitation, bubbles formed flow along the stream. The higher the bubble life, more of the liquid downstream of the orifice is affected by the cavitational effect. The simulation studies provide an idea about the variables that can be manipulated to control the above cavity/bubble behavior for maximum cavitational effects.

(1) A rise in the discharge pressure and hence the final recovery pressure will result in an increase in the active cavitational volume downstream of the orifice with increased cavitational intensity.

(2) The pipe size downstream of the orifice does provide another means of control over cavitational intensity and active volume.

(3) The orifice to pipe diameter ratio provides a possible means of control over cavitationally active volume without changing the cavitational intensity.

(4) Although there is a wide variation in the initial bubble sizes that are naturally generated downstream of the orifice, the contribution of small bubbles to the cavitational effect observed in hydrodynamic cavitation reactor appears to be significant.

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